MIXING MODEL FOR THERMAL ENERGY STORAGE WATER TANK OF MULTI-CONNECTED MIXING TYPE

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1. INTRODUCTION
Water thermal storage tank of multi-connected mixing type is one of the most popular thermal storage tanks in Japan and is introduced in AHSRAE handbook as a labyrinth tank (ASHRAE, 1995). The water storage tanks use effectively spaces between footing beams of building base, therefore the construction cost of tanks is relatively low. The thermal energy storage tank is generally consisting of more than 15 tanks connected in series with connecting holes due to inhibit the mixing of lower temperature water (e.g. chilled water from refrigerator) and higher temperature water (e.g. return water from AHU). In each tank of multi-connected mixing type, tank water is well mixed and water temperature is uniform. However, this is an ideal case and in practice there are some unused water mass for thermal storage due to short-circuit stream in the tank. The performance of thermal storage tank is reduced by the short-circuit stream in the tank. And the input conditions and connecting holes arrangement and its size have also a great effect on the mixing behavior in the tank. Therefore, the connecting holes are designed to be well mixed water in each tank depending on inflow rate.

In this paper, a mixing model for the thermal storage tank of multi-connected mixing type is presented. A past model in wide general use for a simulation of thermal storage system is based on the assumption that water in each tank is perfectly mixed (Nakahara et al., 1988). The performance of thermal storage tank may be overestimated by using this model. In the mixing model presented in this paper, the vertical temperature distribution in each tank is taken into consideration. And an existence of unused water mass for thermal storage in the tank can be found by system simulations using the mixing model. The mixing model is based on the mixing mode for temperature-stratified thermal storage tank and is applicable to the thermal storage tank of both temperature-stratified type and multi-connected mixing type.

Some experiments were conducted using an experimental storage tank. The experimental tank is 1/3 scale model to general actual tank. The calculated results by using the mixing model are compared with the experimental results.

2. MODELING OF MIXING BEHAVIORE IN TANK
The mixing model presented in this paper, is based on the mixing model for the temperature-stratified thermal storage tank (Kitano et al., 2003). It is assumed that vertical one-dimensional diffusive and convective heat transfer is considered and inflow water from inlet pipe or connecting hole is mixed with the tank water in a region according
to input conditions and vertical temperature distribution in the tank (Figure 1).

Equation (1) is the governing equation of the mixing model in this paper.

\[
\frac{\partial T_{st}}{\partial t} = \kappa_0 \frac{\partial^2 T_{st}}{\partial z^2} - U_{st} \frac{\partial T_{st}}{\partial z} + \frac{\Phi}{A_{st}} (T_{in} - T_{st})
\]

(1)

where \(U_{st}\): velocity in horizontal cross section of the tank [m/s], \(T_{st}\): water temperature in the tank [°C], \(\kappa_0\): thermal diffusivity of water in thermal storage tank [m²/s], \(T_{in}\): inflow water temperature from connecting hole or inlet pipe [°C], \(\Phi\): inflow rate per unit vertical depth in the mixing region [m²/s], \(t\): time [s], \(z\): height from the bottom of tank [m] and \(A_{st}\): horizontal sectional area of the tank. \(\Omega\) is outflow rate per unit vertical depth [m²/s] and the integrated value of \(\Omega\) or \(\Phi\) from 0 to \(L_{st}\) is equal to flow rate, \(q_{in}\). And \(\Phi\) and \(\Omega\) is formalized as equation (3) and equation (5) and \(D\) in equation (3) is expressed by equation (4).

\[
q_{in} = \int_{0}^{L_{st}} \Omega \, dz = \int_{0}^{L_{st}} \Phi \, dz
\]

(2)

\[
\Phi = \frac{(z_{ineq} - l_{m1})^2 - (z - z_{ineq})^2}{D} q_{in} \quad \text{if} \quad z_{in,h} - l_{m1} < z < z_{ineq}
\]

\[
\Phi = \frac{(z_{ineq} - l_{m1})^2 - (z_{ineq} - l_{m1})^2}{D} q_{in} \quad \text{if} \quad z_{ineq} < z < z_{in,h} + l_{m2}
\]

(3)

\[
D = \int_{z_{ineq} - l_{m1}}^{z_{ineq} - l_{m1}} (z_{ineq} - l_{m1})^2 - (z - z_{ineq})^2 \, dz + \int_{z_{ineq} - l_{m1}}^{z_{ineq} + l_{m2}} (z_{ineq} - l_{m1})^2 - (z_{ineq} - l_{m1})^2 \, dz
\]

(4)

\[
\Omega = \frac{4d_{e,out}^2 - 4(z_{m,bot} - z)^2}{\pi d_{e,out}^2} q_{in}
\]

(5)

where \(l_{m1}\) is a distance from the the centroid of inflow to the bottom of tank, \(l_{m2}\) is a distance from the the centroid of inflow to the water surface in the tank, \(z_{in,h}\) is the distance of centroid of inflow from the tank bottom [m] and \(d_{e,out}\) is the apparent diameter of virtual outlet pipe [m].

The depth of mixing region is divided into two parts and these depths are calculated by equation (6) obtained as an experimental equation. The experimental equation is obtained from experimental results of temperature-stratified thermal storage tank under various conditions.

\[
l_{m,j} = 0.8 Ar_{m,j}^{-0.5} d_{in} \quad (j = 1, 2)
\]

(6)
\[ Ar_{in,j} = g d_{in} \frac{\Delta \rho_j}{\rho_{r,j}} \frac{1}{u_{in}^2} \quad (j = 1, 2) \]  

where \( Ar_1 \) and \( Ar_2 \) are the Archimedes number, which are defined by equation (7) using inflow velocity, \( u_{in} \), from inlet pipe or connecting holes, the reference density of tank water, \( \rho_{r,1} \) and \( \rho_{r,2} \), and the difference between the density of inflow water and the reference density of tank water, \( \Delta \rho_1 \) and \( \Delta \rho_2 \). \( Ar_1 \), \( \rho_{r,1} \) and \( \Delta \rho_1 \) are variables concerned with the lower part of mixing region, and \( Ar_2 \), \( \rho_{r,2} \), and \( \Delta \rho_2 \) are variables concerned with the upper part of mixing region.

The reference density of tank water is determined according to the following procedure: It is presupposed that the vertical water jet flows into the tank from the centroid of inflow toward the bottom of tank or the water surface in the tank, and water density of the jet is equal to input water density into the tank and the velocity of jet at \( l_j = 0 \) is also equal to the one of input water. And the vertical velocity of this water jet at the distance from inflow centroid, \( l_j \), is assumed to be expressed by the following equation (8).

\[ \rho_{in} u_{pj}^2 = \rho_{in} u_{in}^2 + (-1) \int_0^{l_j} (\rho - \rho_{in}) g \, d \, l_j \quad (j = 1, 2) \]  

where \( l_j \) is the vertical distance from the inflow centroid [m], \( \rho_{in} \) is the inflow water density [kg/m\(^3\)], \( \rho \) is tank water density at which the vertical distance from the inflow centroid is \( l_j \) [m], \( u_{pj} \) is the vertical velocity of water jet [m/s], \( g \) is the acceleration of gravity [m/s\(^2\)] and \( u_{in} \) is the inflow velocity [m/s].

The vertical position where the water jet reaches is obtained by using equation (8). The reference temperature is defined that the water temperature at the depth where the jet reaches, and the reference density in the tank is defined as the density of water at the reference temperature. When the water jet reaches to the bottom or water surface of the tank, the reference density is the water density at the bottom or water surface of the tank.

The distribution of inflow to the mixing region, \( \Phi \), is determined as follows. The inflow rate per unit depth, \( \Phi \), is expressed by quadratic functions and it has the maximum value at the position where the water temperature is equal to the inflow water temperature. And the distribution rate of inflow water is equal to zero at both ends of the mixing region. When the mixing region is beyond the water surface or the bottom of tank, the distribution of inflow water is truncated at the water surface or the bottom of tank.

The flow rate distribution of outflow, \( \Omega \), is determined as follows. It is assumed that outflow from the tank has uniform velocity in cross section of virtual outlet pipe. And the diameter of virtual outlet pipe is determined according to the outflow velocity, the diameter of outlet pipe and outlet Archimedes number by using the following experimental equation.

\[ d_{e,out} = 1.4 d_{out} Ar_{out}^{-0.24} \]  

\[ Ar_{out} = g d_{out} \frac{\Delta \rho_{out}}{\rho_{r,out}} \frac{1}{u_{out}^2} \]  

where \( d_{out} \) is the diameter of outlet pipe [m], \( d_{e,out} \) is the apparent diameter of virtual outlet pipe [m] and \( Ar_{out} \) is an outlet Archimedes number defined by equation (10). \( \rho_{r,out} \) is the reference density of tank water and \( \Delta \rho_{out} \) is the difference between the reference density of tank water and the density of the tank water at the same vertical position of outlet pipe center. The reference density of tank water is the higher density of tank water at the lower and upper end of outflow region. The outlet Archimedes number is dependent on the apparent diameter of outlet pipe, therefore, the apparent diameter is obtained by iterative calculations.

### 4. COMPARISON OF EXPERIMENTAL RESULTS WITH CALCULATED RESULTS

Experiments were conducted using scale model tanks to clarify the mixing behavior in the multi-connected mixing type thermal storage tank and to verify the mixing model. The experimental tanks are shown in figure 2. The experimental tank consist of four tanks connected in series, and is designed as approximately 1/3 scale model of actual thermal storage tank. The experiments are conducted by using the experimental tank with two pattern of
connecting holes arrangement. One is the arrangement which connecting holes are placed at the lower part of the tank in series, and the other is the arrangement which connecting holes are set alternately lower and upper part in the tank. In the experiments, vertical temperature distribution in each tank, connecting holes and inlet pipe are measured. The position of the thermocouple installed is shown in figure 2. Table 1 shows the experimental conditions which results are presented in this paper.

Table 1: Experimental conditions

<table>
<thead>
<tr>
<th>No.</th>
<th>Duration of experiment [h]</th>
<th>Input water temperature [°C]</th>
<th>Initial temperature [°C]</th>
<th>Flow rate [m³/h]</th>
<th>Archimedes number*</th>
<th>Connecting holes arrangement type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.2</td>
<td>15.7 - 10.0</td>
<td>27.1</td>
<td>2.68</td>
<td>2.087</td>
<td>Type A</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>25.0 - 22.0</td>
<td>27.9</td>
<td>9.27</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.9</td>
<td>19.4 - 20.5</td>
<td>8.7</td>
<td>2.00</td>
<td>2.197</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3.0</td>
<td>11.7 - 8.3</td>
<td>24.9</td>
<td>2.01</td>
<td>3.663</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.5</td>
<td>21.8 - 20.3</td>
<td>25.9</td>
<td>6.89</td>
<td>0.124</td>
<td>Type B</td>
</tr>
<tr>
<td>6</td>
<td>5.8</td>
<td>9.8 - 7.9 (0.0 - 2.6 h)</td>
<td>23.9</td>
<td>2.04</td>
<td>3.424</td>
<td>Type A</td>
</tr>
</tbody>
</table>

* : This Archimedes number is defined as the following equation. In the equation, \( \rho_0 \) is water density at initial temperature in the tank, \( \Delta \rho \) is the density difference between \( \rho_0 \) and water density at average input water temperature while elapsed time is from 0.25 to 0.50 hours, \( u_c \) is water flow velocity at the connecting hole and \( d_c \) is the diameter of the connecting holes.

\[
Ar = g \frac{d_c \Delta \rho}{\rho_0 u_c^2}
\]

Figure 2: Experimental tank
The equations of the mixing model are solved by an explicit finite differential method. The tank water is vertically divided into 200 and time increment is 0.1 seconds. In this calculation, the vertical temperature distributions in the tank were calculated by using the initial temperature in the tank and the input conditions measured during each experiment.

<table>
<thead>
<tr>
<th>Time [h]</th>
<th>Inflow water temperature [°C]</th>
<th>Flowrate [m³/h]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
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<td>1.0</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Inflow water temperature
Flow rate

Figure 3: Comparison of the experimental results and the calculated results in the case that connecting holes arrangement is Type A (refer to figure 2)
Figure 4: Comparison of the experimental results and the calculated results in the case that connecting holes arrangement is Type B

Figure 5: Comparison of the experimental results and the calculated results in the case that connecting holes arrangement is Type A (Experiment No. 6)
Figure 3 shows experimental results and calculated results in the case that the connecting holes arrangement is Type A (refer to figure 2). Left figure shows the inlet water temperature into the 1st tank and flow rate, and right figure shows the temperature distribution change in each tanks. In the case that input water temperature was lower than tank water temperature, it was found that water in the upper part of the experimental tanks (the 2nd and 3rd tank in figure 3(a)) was not mixed well with the inflow water from the connecting holes and remained in high temperature. The tank water in this region is not used for the thermal storage and therefore heat capacity of the thermal storage tank is decreased. When the Archimedes number was higher (figure 3(b)) or inflow water temperature was higher than tank water temperature (figure 3(c)), water in each tank was mixed well. There were some differences in the temperature gradient at stratified region and the position of stratification in figure3 (a). These results show a necessity of a few corrections for the experimental equation (equation (4)). However, the calculated results on temperature profiles in the tanks agree with the experimental results.

Figure 4 shows the experimental results and calculated results in the case that the connecting holes arrangement is Type B (refer to figure 2). In this connecting holes arrangement, the water in each tank was mixed well with the inflow water even if the Archimedes number was relatively high as in figure 4(a). The calculated water temperature at the upper part of the 1st tank was lower than the measured temperature in the experiment No.4. It was also found that the calculated results agree with the experimental results under the conditions of this connecting holes arrangement.

Figure 5 shows the results under the conditions that the inflow water temperature was changed from lower to higher than the tank water. Under the conditions which Archimedes number was relatively high and the connecting holes were arranged near bottom in series, most of tank water was not effectively used for thermal storage in the 2nd tank and the 3rd tank shown in figure 5. It is found that the calculated results were also agreement with the experimental results under this input condition.

5. CONCLUSION

In this paper, a mixing model which can be applied to multi-connected mixing type thermal storage tank is presented. And the experiments carried out in the model tank which consists of four tanks connected in series. When the connecting holes were set at the bottom of tank partition in series, the upper part of each tank was not used effectively for thermal storage. When the connecting holes were arranged upper and lower alternately, the tank water tended to be mixed well even if velocity of inflow was relatively low. And the temperature profiles in the tank calculated by using the mixing model were compared with the results of experiments. It was found that the calculated results by using the mixing model agreed with the experimental results.

REFERENCES

